Verti-zontal Differentiation in Monopolistic Competition*

Francesco Di Comite[†] Jacques-François Thisse[‡] Hylke Vandenbussche[§]

Preliminary version - Please do not quote without permission

March 11, 2011

This paper presents a model of monopolistic competition where the observed heterogeneity in sales, profits and markups across firms can be explained by differences in quality, costs and consumer tastes. The standard quadratic utility function is generalized to allow for demand parameters capturing variety-specific vertical differentiation and market-variety-specific horizontal differentiation. In each market, prices are shown to depend on aggregate quality and productive efficiency, through the effective mass of competitors and the degree of substitutability across varieties. The interaction between supply- and demand-side heterogeneity allows us to accommodate recent empirical evidence based on micro-level trade data and obtain results that models of cost or quality heterogeneity alone are unable to capture. In addition, we uncover the possibility to break the univocal relation between markups and sales, for given levels of quality, which plagues most models of monopolistic competition, thus providing a more flexible theoretical framework for dealing with micro-level trade data.

JEL codes: D43 - F12 - F14 - L13 - L16 - R32

Keywords: Heterogeneous firms - Product Differentiation - Monopolistic Competition - Market Indices - Market Integration.

---

*We thank Andrew Bernard, Giovanni Peri, Kaz Miyagiwa, Davide Castellani, Xavier Wauthy for helpful discussions and suggestions. We are grateful to Florian Mayneris and Mathieu Parenti for detailed reading and commenting on earlier drafts. We thankfully acknowledge financial support from the Belgian French-speaking Community (convention ARC 09/14-019 on "Geographical Mobility of Factors").

†Corresponding author. Department of Economics, Université Catholique de Louvain, Place Montesquieu 3, 1348 Louvain-la-neuve, Belgium - Email: francesco.dicomite@uclouvain.be - Tel: +32 (0) 10 473434

‡CORE, Université Catholique de Louvain, Voie du Roman Pays 34, 1348 Louvain-la-neuve, Belgium - Email: jacques.thisse@uclouvain.be - Tel: +32 (0)10 474312

§CORE and Department of Economics, Université Catholique de Louvain, Place Montesquieu 3, 1348 Louvain-la-neuve, Belgium - Email: hylke.vandenbussche@uclouvain.be - Tel: + 32 (0)10 474137

1 Introduction

It is today uncontroversially recognized that firms are heterogeneous along different dimensions, even within the same sector and geographical market. While empirical evidence corroborating this claim abound, an encompassing theoretical framework making full sense of this multi-dimensional heterogeneity and its implications is still wanting.

Early attempts to model heterogeneity focused on differences in productive efficiency within a demand system exhibiting constant elasticity of substitution (CES) in order to fit the empirical evidence available at the time in an elegant and parsimonious way (Melitz, 2003). However, careful scrutiny of the properties of such models shows that, despite their relevance at an aggregate level, they fail to account for several empirical regularities at more disaggregate levels of analysis. For example, heterogeneity in costs alone cannot account for exporters charging higher domestic prices than non-exporters,[1] while the observed price discrimination across markets cannot be explained by standard CES specifications.[2] Recent contributions reacted to these drawbacks by exploiting different demand specifications in order to have non-constant markups and richer interactions between firms and competitive environments (Melitz and Ottaviano, 2008), or by introducing additional dimensions of heterogeneity, the most common being quality.[3] This paper merges these two approaches by nesting multi-dimensional heterogeneity in the quadratic utility framework proposed by Ottaviano et al. (2002). To the best of our knowledge, only four papers currently share these characteristics, Antoniades (2008), Kneller and Yu (2008), Foster et al. (2008) and Altomonte et al. (2010), their demand specifications being special cases of the one developed here.

Few empirical papers have tried to explicitly take into account vertical and horizontal differentiation in trade and the ones we are aware of are all empirical works based on models of discrete choice (Anderson et al., 1992), where horizontal differentiation is mainly interpreted as a random demand shifter affecting both prices and quantities.[4] Anyway, a common feature in this strand of literature is that quality and marginal costs alone do not suffice to have a full characterization of how a product performs in a market. Horizontal differences in consumer taste seem to play an important role too, as clearly suggested by the common presence of a "home bias effect" in the widest range of contexts, ranging from car markets (Goldberg and Verboven, 2005) to the wine sector (Brooks, 2003; Friberg et al., 2010) to cultural industries (Chung and Song, 2008; Ferreira and Waldfogel, 2010).

[1] The claim that "exporters are different" (Bernard and Jensen, 1994) has been confirmed by Johnson (2007), Iacovone and Javorcik (2008), Crozet et al. (2009) and Kugler (2008).

[2] See, for example, Fontagné et al. (2008), Hallak and Sivadasan (2009), Goldberg and Verboven (2001), Gorg et al. (2010) and Schott (2004).

[3] Notable examples of quality-augmented CES models are Fajgelbaum et al. (2009); Gervais (2008); Hallak and Sivadasan (2009); Helble and Okubo (2008); Johnson (2007) and Kranich (2007).

[4] Recent examples are Katayama et al. (2009); Khandelwal (2009) and Verhoogen (2008).

Our paper aims at responding to these empirical challenges by combining the inputs provided by different strands of literature to build a model of monopolistic competition in which idiosyncratic elements of vertical and horizontal product differentiation determine markups, sales and market characteristics. The starting point is the increasingly popular quadratic utility function. In its original specification, demand parameters are meant to apply uniformly to all the varieties in a particular sector. In our case, they are allowed to differ across varieties in the same sector. This provides new insights on the mechanisms driving the original results and offers an encompassing theoretical framework for empirical studies. In our model, demand for each variety is characterized by three elements: (i) a pure demand shifter affecting prices and quantities, which will be regarded as capturing the vertical dimension of differentiation; (ii) an output shifter affecting only the slope of the demand function, leaving own prices unaffected; (iii) a substitutability parameter capturing the degree of indirect competitive pressure exerted by the other varieties.

With this rich parametrization, product prices may range from pure monopoly to barely covering marginal costs of production. The possibility of even lower prices is excluded, as firms are assumed to leave the market rather than producing at a loss. Quantities sold are directly related to markups as in the standard quadratic utility model, but the ratio of this relation is now allowed to change for each variety depending on its specific horizontal attributes. In different markets the same variety may then be sold at different prices and in different quantities, even when the differences in costs are negligible. This market-specific degree of competitiveness can be fully captured by taste-weighted price, quality and cost indices, in addition to the effective mass of competitors in the market. Product characteristics affect quantities sold of each variety in such a way that varieties matching well local tastes are weighted more in these indices than varieties ignored by local consumers. Fortunately, what we call "taste," for the sake of simplicity, need not be directly observed, but can be indirectly captured by a horizontal differentiation parameter, which measures the mismatch between the consumers' ideal and actual product characteristics. Therefore, as a byproduct of this model, new market indices emerge for measuring the competitiveness of a market, which may complement the ones presented by Gaulier et al. (2008).

Admittedly, many of the empirical facts that can be captured by the model presented in this paper have been individually addressed by other empirical and theoretical papers. Our purpose is to propose a general model of monopolistic competition being flexible enough to embrace all the results of previous intra-industry trade models, yet remaining intuitive, tractable and empirically identifiable.

To sum up, our generalization of the quadratic utility function may comply with both vertical and horizontal differentiation, while allowing for differences in quantities consumed for each variety. We will see that our setting captures most of the main effects emphasized by game-theoretic models of product differentiation (Tirole, 1988), yet retains the analytical flexibility that features monopolistic competition. Unlike industrial organization models that emphasize strategic interactions between firms, our approach focuses on "weak interactions " between firms, meaning that firms' behavior is influenced only by market aggregate statistics which are themselves unaffected by the choices made by any single firm. We should also add that industrial organization models developed to deal with multidimensional heterogeneity are analytically hard to handle (see, for example, Irmen and Thisse, 1998). Thus, we find it fair to say that our setting provides a reconciliation of the main ideas and results developed in industrial organization with the recent micro-level approaches used in empirical studies developed in the international trade literature.

The remaining of the paper is organized as follows. The next section presents more systematically the body of evidence motivating the model, section 3 presents the model and its properties, section 4 concludes.

2 Stylized Facts

In this section we report some empirical facts challenging the state of the art in intra-industry trade modeling based on monopolistic competition. The distinct role of quality, taste and substitutability in reconciling theory and empirics is discussed and the body of evidence against which current theoretical modeling must be addressed is sketched out.

As the increasing availability of micro-level data allows to test implications and results of new trade models, early attempts to model heterogeneity through cost differences (Melitz, 2003; Melitz and Ottaviano, 2008) have recently undergone intense scrutiny. One directly testable implication of models of cost heterogeneity is that, for a given demand, higher prices should be associated with lower profits and quantities sold. Authors testing explicitly for this implication (Crozet et al., 2009; Hummels and Klenow, 2005; Kugler, 2008; Manova and Zhang, 2009) tend to reject it and suggest that additional dimensions of heterogeneity are needed.

The usual suspect as a relevant source of additional variability is generally referred to as quality.[5] As Hallak and Sivadasan (2009) point out, the interaction between product quality and productive efficiency may help explain the "*empirical fact that firm size is not monotonically related with export status*" and that "*there are small firms that export while there are large firms that only operate in the domestic market.*" Also Brooks (2006) proposes a similar explanation to account for the "*tendency for Colombian plants to under-export manufactured goods to the United States.*" In general, it seems reasonable to expect that quality and efficiency have a different relative weight depending on the industry, as Kneller and Yu (2008) suggest by looking at Chinese exporters and Khandelwal (2009) confirms by studying US imports.

[5] See, for example, Edwards and Lawrence (2010); Foster et al. (2008); Helble and Okubo (2008); Hummels and Klenow (2005); Iacovone and Javorcik (2008) and Gervais (2010).

As a matter of fact, at a macro level, recent empirical works consistently report that differences in product quality, as generally measured by differences in unitary prices, are an important determinant of the pattern of international trade flows across countries.[6] Anyway, it is important to notice that "*international specialization is taking place within products across varieties, rather than across products or industries*" (Fontagné et al., 2008). Therefore, ignoring idiosyncratic shifts in demand for single varieties within a sector may bias results (Foster et al., 2008; Katayama et al., 2009) and hide rich underlying processes (Kneller and Yu, 2008).

The importance of quality in shifting demand seems to be confirmed also at a micro level by trade data on a wide range of countries, such as China (Manova and Zhang, 2009; Schott, 2008), Colombia (Brooks, 2006; Eaton et al., 2007; Kugler and Verhoogen, 2007) and Mexico (Iacovone and Javorcik, 2008). Furthermore, exploiting product-level Hungarian custom data, Gorg et al. (2010) show that even the same product may be sold at a very different unitary price in different markets, justifying the claim that local competitive pressure may be as important as idiosyncratic quality as a demand shifter. This view is comfortably confirmed to hold also within integrated markets, such as the EU in the case of the car industry (Goldberg and Verboven, 2001), at virtually any geographical scale, even cities within the same country (Engel and Rogers, 2000).

Likely demand shifters such as quality-improving investments (Iacovone and Javorcik, 2008) and high-quality inputs (Kugler, 2008) are indeed consistently and significantly associated with prices and turnover, but do not seem to perform well in predicting output levels. Using plant-level manufacturing data on revenues and physical output and focusing on firms serving local markets, Foster et al. (2008) show that "*a large dispersion in output across producers of the same product*" is observed, even "*after taking into account productivity variations and the movements along the demand curve associated with these.*" The presence of this large unexplained variability may also be contributing to the puzzlingly weak relation between productivity and size (Brooks, 2006; Hallak and Sivadasan, 2009) and to the empirical evidence of a bias towards domestic varieties (Brooks, 2003; Chung and Song, 2008; Ferreira and Waldfogel, 2010; Goldberg and Verboven, 2005). No theoretical model, to our knowledge, takes advantage of the information provided by output variability to adjust price indices and other relevant measures of competition, which is one key contribution of this paper.

Besides technical efficiency, product quality and taste-driven idiosyncratic output variations, another commonly envisaged variable affecting firms' performance appear to be product substitutability within and across sectors. Khandelwal (2009), for example, claims that there is "*substantial heterogeneity in product markets' scope for quality differentiation*" relating within-sector competition between varieties to the "length" of the quality ladder. According to the author, this could explain the heterogeneity across in-

[6] Relevant works directly addressing this issue are Baldwin and Harrigan (2007); Hallak (2004); Hummels and Klenow (2005); Hummels and Skiba (2004) and Schott (2004).

dustries in the impact of low-wage competition on U.S. output and employment growth reported by Bernard et al. (2006). Similarly, using data on geographically isolated monopolies, duopolies, and oligopolies in retail and professional industries, Bresnahan and Reiss (1991) find important inter-industry differences in entry threshold ratios, suggesting that the patterns of substitutability vary across markets. Finally, running a principal component analysis on the responses provided by owners of London-based businesses to a set of questions about their priorities, Gordon (2010) notes that, together with quality and efficiency, differentiation stands out as an important strategic dimension of competition, suggesting that firms do take indirect competition into account when setting their pricing strategies and try to relax it by horizontally differentiating their offer within a sector as well.

3 Re-thinking product differentiation in monopolistic competition

In this section, we introduce various types of heterogeneity in the quadratic utility model with the aim to find out whether such changes may replicate some of the main empirical facts discussed above. Although our ultimate objective is to compare variables across different markets, at this stage we find it notationally convenient to focus on one market, thus avoiding to index it.

There are several definitions of vertical and horizontal differentiation, which are (more or less) equivalent. Ever since Hotelling (1929), two varieties of the same good are said to be horizontally differentiated when there is no common ranking of these varieties across consumers. In other words, horizontal differentiation reflects consumers' idiosyncratic tastes. By contrast, two varieties are vertically differentiated when all consumers share the same ranking. Vertical differentiation thus refers to the idea of quality intrinsic to these varieties (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982). Such definitions of horizontal and vertical differentiation have been proposed for indivisible varieties with consumers making mutually exclusive choices. In what follows, we generalize these definitions in two directions: we allow (i) the differentiated good to be divisible and (ii) consumers to buy more than one variety. [7]

3.1 The one-variety case

The economy involves one differentiated good and one homogeneous good, which is used as the numéraire. There is one consumer who is endowed with income y. Consider one variety s of the differentiated good. The utility from consuming the quantity $q_s > 0$ of

[7] Note that our approach, like most models of monopolistic competition, abstracts from the way product characteristics are chosen by firms. This issue has been tackled in a handful of theoretical papers (Hallak and Sivadasan, 2009; Neven and Thisse, 1989) and analyzed empirically by Kneller and Yu (2008) and (Kugler and Verhoogen, 2007).

this variety and the quantity $q_0 > 0$ of the numéraire is given by

$$u_s = \alpha_s q_s - \frac{\beta_s}{2} q_s^2 + q_0$$

where $\alpha_s > 0$ and $\beta_s > 0$. The budget constraint is

$$p_s q_s + q_0 = y$$

where p_s is the price of variety s. Plugging the budget constraint in u_s and differentiating with respect to q_s yields the inverse demand for variety s:

$$p_s = \max\left\{\alpha_s - \beta_s q_s, 0\right\}. \tag{1}$$

In this expression, p_s is the highest price the consumer is willing to pay to acquire the quantity q_s of variety s, i.e. her willingness-to-pay (WTP). When the good is indivisible, the WTP is constant. Here, instead, it declines with consumption, following the decrease in its marginal utility. The WTP is positive provided that q_s is smaller than α_s/β_s. When q_s exceeds this value, the numéraire becomes more attractive than one additional unit of variety s.

Consider now any two varieties, $s = 1$ and $s = 2$. Intuitively, these two varieties are vertically differentiated if the consumer views the vertical characteristics of variety 1 as dominating those of variety 2. We have just seen that the WTP for a variety decreases with its consumption level. Therefore, when the consumption of each variety is variable, we say that varieties 1 and 2 are vertically differentiated when all consumers' WTP for the first marginal unit of variety 1 exceeds that of variety 2, i.e. $\alpha_1 > \alpha_2$. The increase of WTP in α_s thus implies that this parameter captures the vertical characteristics embodied in the differentiated product. Consequently, it seems natural to interpret α_s as a measure of the "quality" of variety s. An alternative definition would be to say that varieties 1 and 2 are vertically differentiated when $\alpha_1 - \beta_1 q > \alpha_2 - \beta_2 q$ for all $q > 0$. However, this definition overlaps with the definition of the WTP that captures more features than vertical attributes only. For example, we find it natural to expect the WTP of a variety to depend on its horizontal attributes.

Solving for q_s, we obtain the demand function for variety s:

$$q_s = \max\left\{\frac{\alpha_s - p_s}{\beta_s}, 0\right\}$$

which is positive if and only if α_s exceeds p_s. As long as the WTP for one additional unit of variety s is positive, a consumer chooses to acquire more of this variety. In contrast, she chooses to consume more of the numéraire when the WTP is negative. The equilibrium consumption is obtained when the WTP is equal to zero.

We now assume that the differentiated good is available in a continuum $S \equiv [0, N]$ of varieties, where N is the mass of varieties. The utility of variety s is now given by

$$u_s = \alpha_s q_s - \frac{\beta_s}{2} q_s^2 - \frac{\gamma}{2} q_s \left[\int_S q_r dr \right] + q_0$$

$$= \alpha_s q_s - \frac{\beta_s}{2} q_s^2 - \frac{\gamma}{2} q_s Q + q_0 \tag{2}$$

where $\gamma > 0$ and Q is the total amount of the differentiated good consumed. In this expression, γ measures the substitutability between variety s and any other variety $r \in S$. Stated differently, all varieties enter symmetrically into the utility function.

In (2), $\alpha_s - \gamma Q/2$ is the marginal utility derived from consuming the first unit of variety s. It varies inversely with the total consumption of the differentiated product because the consumer values less variety s when Q is larger. Note that the intercept is positive provided that the desirability of variety s (α_s) dominates the negative impact of the total consumption of the differentiated product weighted by the degree of substitutability across varieties (γ). As q_s increases, the marginal utility of this variety decreases and the equilibrium consumption of variety s is reached when its marginal utility, which depends on Q, equals the marginal utility of the numéraire.

Repeating the above argument, the WTP of variety s becomes

$$p_s = \alpha_s - \frac{\gamma}{2} Q - \beta_s q_s. \tag{3}$$

Compared to (1), the WTP for variety s is shifted downward to account for the fact that

all varieties are substitutes; the value of the shifter increases with the total consumption of the differentiated good but decreases with the degree of product differentiation.

We now come to the interpretation of parameter β_s. In the hope of making this interpretation more transparent, we appeal to the Hotelling spatial metaphor wherein the economy is populated by a continuum of heterogeneous, but *fictituous*, consumers. By the same token, we will illustrate the differences between Hotelling's model, where consumers make mutually exclusive choices, and our model in which the consumer is allowed to acquire bundles of varieties. As in spatial models of product differentiation, we assume that, provided that its WTP is positive, the demand for a variety is perfectly inelastic and equal to $\hat{q}$ that need not be equal to one.

In Figure 1, we depict a symmetric setting with varieties/shops 1 and 2 located at the endpoints of a unit segment, where $\alpha_1 = \alpha_2 = \alpha$ and $\beta_2 = 1 - \beta_1 > 0$. As in the

Hotelling metaphor, this segment describes consumers' location. Using (3), the WTP for, say, variety 1 has an intercept equal to $\alpha - \gamma\hat{q}/2$ and decreases, at a "transport rate " equal to $\hat{q}$, while the distance between variety 1 and the consumer is given by β_1. The consumer's WTP for variety 1 equals zero at

$$\beta_{\max} = \alpha/\hat{q} - \gamma/2.$$

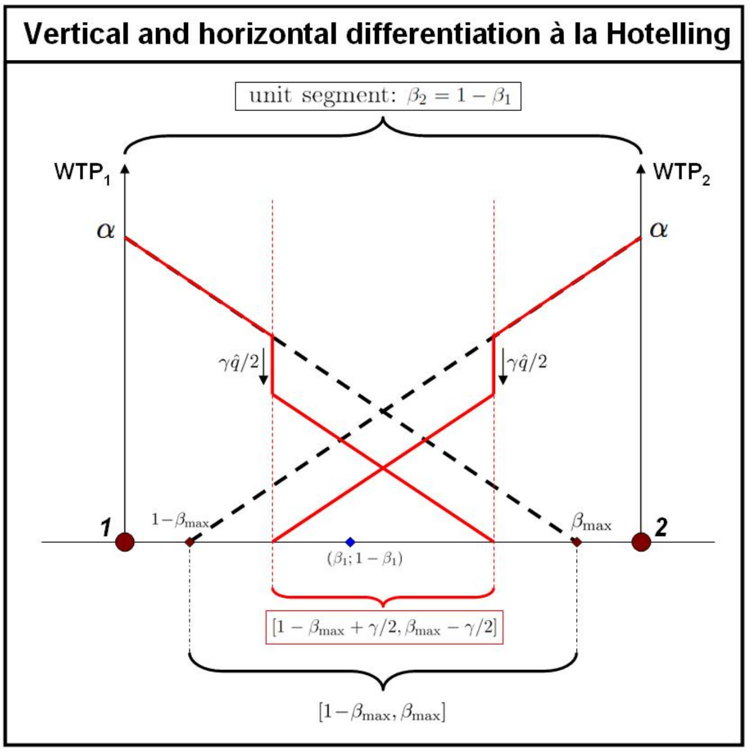

Figure 1: Graphical intuition of the spatial problem

Treading in Hotelling's footsteps, when the consumer is located at $\beta_1 \in [0, \beta_{\max}]$, she is willing to consume the quantity $\hat{q}$ of variety 1 because her utility remains positive as long as the distance to shop 1 is smaller than $\beta_{\max}$. Therefore, a high (low) value of β_1 amounts to saying that the consumer is far from (close to) shop 1. As a result, we may view β_s in (2) as a parameter expressing the *idiosyncratic mismatch* between the horizontal characteristics of variety s and the consumer's ideal. At this stage, we find it fair to say that the preferences (2) encapsulates both vertical (α_s) and horizontal (β_s) differentiation features. How to relate this new interpretation of β_s to the concavity degree of u_s? As the mismatch between variety s and the consumer's ideal horizontal characteristics β_s increases, it seems natural to expect the consumer to reach faster the level of satiation. In other words, if our consumer prefers vanilla to chocolate as an ice-cream flavor, the utility of an additional chocolate scoop will decrease faster than that of a vanilla scoop.

We now proceed by exploring the links between the Hotelling setting and our model of monopolistic competition. When $\beta_1 < \beta_{\max}$, we know that the consumer patronizes at least shop 1. However, as long as $\alpha - \gamma\hat{q}/2 - \beta\hat{q}$ is positive at 1/2, then there is another segment $[1 - \beta_{\max}, \beta_{\max}]$ in which both $\alpha - \gamma\hat{q}/2 - \beta_1\hat{q}$ and $\alpha - \gamma\hat{q}/2 - (1 - \beta_1)\hat{q}$ are positive. This suggests that the consumer located in the vicinity of 1/2 may want to visit both shops. In this event, however, the total quantity of the good is no longer equal to $\hat{q}$; instead, it is equal to $2\hat{q}$. This in turn implies that the two WTP-lines shift downward by $\gamma\hat{q}/2$. Therefore, the segment over which both shops are actually patronized is narrower and given by $[1 - \beta_{\max} + \gamma/2, \beta_{\max} - \gamma/2]$. Consequently, when the consumer located at $\beta_1 < 1 - \beta_{\max} - \gamma/2$ she visits shop 1 only, whereas she visits both when her location belongs to the interval $[1 - \beta_{\max} + \gamma/2, \beta_{\max} - \gamma/2]$.

The foregoing argument shows how the Hotelling model can be extended to cope with consumers buying different varieties of the differentiated good. [8] In particular, regardless of her location β_1, a consumer acquires the two varieties when the interval $[1 - \beta_{\max} + \gamma/2, \beta_{\max} - \gamma/2]$ is wide enough to include the unit segment. This will be so if and only if

$$\alpha - \gamma\hat{q} > \hat{q}$$

that is, the intercept of the WTP exceeds the fixed requirement of a single variety. This is likely to hold when the desirability of the differentiated good is high or the substitutability between the two varieties is low. Similarly, when the fixed requirement $\hat{q}$ is small enough, the two WTP lines are almost flat so that any consumer is likely to acquire the two varieties, in order to compensate the low consumption of each.

Conversely, for any fixed amount of consumption $\hat{q}$, it is readily verified that, regardless of her location, the representative consumer acquires a single variety if and only if

$$\hat{q} > 2(\alpha - \gamma\hat{q}) \Leftrightarrow \gamma > \frac{\alpha}{\hat{q}} - \frac{1}{2}.$$

Once varieties are sufficiently good substitutes, all consumers choose to behave as in the Hotelling model, where they are assumed to buy a single variety. Note, finally, that consumers located near the ends of the segment buy only one variety and consumers located in the central area buy both if and only if

$$\alpha - \gamma\hat{q} < \hat{q} < 2(\alpha - \gamma\hat{q}).$$

Summing up, our specification of preferences appears to be broad and flexible enough to encompass various facets of product differentiation.

[8] If β_s is uniformly distributed over the unit segment, then the corresponding population of consumers is identical to the population located over $[0, 1]$. As long as firms can price discriminate across locations, the fictituous consumers becomes actual consumers (Anderson and Neven (1989); Lederer and Hurter (1986)).

3.2 The multi-variety case

Let S be the set of available varieties. Integrating (2) over S yields the utility function

$$U = \int_S \alpha_s q_s ds - \frac{1}{2}\int_S \beta_s q_s^2 ds - \frac{\gamma}{2}\left[\int_S q_s ds\right]^2 + q_0$$

where α_s and β_s are now two positive and continuous functions defined on S, while γ is still a positive parameter that measures the substitutability between any pair of varieties. The above expression is to be contrasted to the standard quadratic utility in which α and β are identical across varieties. As usual in monopolistic competition models, the consumer is free to choose the quantity of each variety she wants to acquire.

The budget constraint is

$$\int_S q_s p_s ds + q_0 = y.$$

Using (3), we readily see that the demand for variety s is given by

$$q_s = \frac{\alpha_s - p_s}{\beta_s} - \frac{\gamma(\mathbb{A} - \mathbb{P})}{\beta_s(1 + \gamma\mathbb{N})} \tag{4}$$

where

$$\mathbb{N} \equiv \int_S \frac{dr}{\beta_r} \qquad \mathbb{A} \equiv \int_S \frac{\alpha_r}{\beta_r} dr \qquad \mathbb{P} \equiv \int_S \frac{p_r}{\beta_r} dr.$$

Like in most models of monopolistic competition, the demand for a variety depends on a few statistics, here three (Vives, 2001). Using the interpretation of β_r given above, it is straightforward to see $1/\beta_r$ as a measure of the proximity of variety r to the representative consumer's ideal set of characteristics. Consequently, a variety having a large β_r has a weak impact on the demand for variety s because the representative consumer is not willing to buy much of it.[9] In contrast, a variety with a small β_r has a strong impact on the consumption of variety s because the representative consumer highly values its horizontal characteristics. All of this explains why β_r appears in the denominator of the three statistics.

Having this in mind, although N is the actual mass of varieties, it should be clear why each one is weighted by the inverse of its taste mismatch to determine the effective mass of varieties, given by $\mathbb{N}$. Indeed, $\mathbb{N}$, and not N, is what the consumer cares about when she chooses how much to consume of a given variety because $\mathbb{N}$ accounts for idiosyncrasies. For example, adding or deleting varieties with bad matches does not affect much her demand for the others, whereas the opposite holds when the match is

[9]Formally, we should consider an open interval of varieties containing r because the impact of a single variety upon another is zero.

good. Note that $\mathbb{N}$ may be larger or smaller than N according to the distribution of taste mismatches. Similarly, the quality and price of a variety are weighted by the inverse of its taste mismatch to determine the effective quality and price indices. In particular, varieties displaying the same quality (or price) may have very different impacts on the demand for other varieties according to their taste mismatches.

The above discussion shows that it is possible to introduce heterogeneity across varieties on the consumer side in order to generate a large array of new features in consumer demand. In what follows, we call *verti-zontal differentiation* this new interaction of vertical and horizontal characteristics.

3.3 Monopolistic competition under verti-zontal differentiation

When each variety s is associated with a marginal production cost $c_s > 0$, operating profits earned from variety s are as follows:

$$\Pi_s = (p_s - c_s)q_s$$

where q_s is given by (4). Differentiating this expression with respect to p_s yields

$$p_s^*(\mathbb{P}) = \frac{\alpha_s + c_s}{2} - \frac{\gamma(\mathbb{A} - \mathbb{P})}{2(1 + \gamma\mathbb{N})}. \tag{5}$$

The natural interpretation of this expression is that it represents firm s' best-reply to the market conditions. These conditions are defined by the aggregate behavior of all producers, which is summarized here by the price index $\mathbb{P}$. The best-reply function is upward sloping because varieties are substitutable: a rise in $\mathbb{P}$ enables each firm to sell its variety at a higher price. Because each firm is negligible, even though the price index is endogenous, it accurately treats $\mathbb{P}$ parametrically. In contrast, $\mathbb{A}$ and $\mathbb{N}$ are exogenously determined by the distributions of quality and tastes. In particular, a larger effective mass $\mathbb{N}$ of firms makes competition tougher and pushes prices downward. Similarly, when the quality index $\mathbb{A}$ rises, each firm faces varieties having in the aggregate a higher quality, thus making harder the market penetration of its variety.

The natural interpretation of this expression is that it represents firm s' best-reply to the market conditions. These conditions are defined by the aggregate behavior of all producers, which is summarized here by the price index $\mathbb{P}$. The best-reply function is upward sloping because varieties are substitutable: a rise in $\mathbb{P}$ enables each firm to sell its variety at a higher price. Because each firm is negligible, even though the price index is endogenous, it accurately treats $\mathbb{P}$ parametrically. In contrast, $\mathbb{A}$ and $\mathbb{N}$ are exogenously determined by the distributions of quality and tastes. In particular, a larger effective mass $\mathbb{N}$ of firms makes competition tougher and pushes prices downward. Similarly, when the quality index $\mathbb{A}$ rises, each firm faces varieties having in the aggregate a higher

quality, thus making harder the market penetration of its variety.

Integrating (5) over S shows that the equilibrium price index can be expressed in terms of three aggregated indices:

$$\mathbb{P}^* = \mathbb{C} + \frac{\mathbb{A} - \mathbb{C}}{2 + \gamma\mathbb{N}} \tag{6}$$

where the cost index is defined as

$$\mathbb{C} = \int_S \frac{c_r}{\beta_r} dr.$$

In this expression, varieties' costs are weighted as in the above indices for the same reasons. Hence, efficiently produced varieties may have a low impact on the cost index when they have a bad match with the consumer's ideal.

Plugging $\mathbb{P}^*$ into (5), we obtain the (absolute) markup of variety s:

$$p_s^* - c_s = \frac{\alpha_s - c_s}{2} - \mathcal{T}\left(\frac{\mathbb{A} - \mathbb{C}}{2\mathbb{N}}\right) \tag{7}$$

In words, a variety markup is equal to half of its social value minus half of the average social value of all varieties, the second term being weighted by a coefficient that accounts for the toughness of competition, i.e.

$$\mathcal{T} \equiv \frac{\gamma\mathbb{N}}{2 + \gamma\mathbb{N}} \in [0; 1]$$

which depends on the effective mass of firms and the degree of substitutability across varieties. In particular, when $\mathcal{T} \to 1$, only the varieties with the highest social value will survive and will be supplied at their marginal cost. When $\gamma\mathbb{N}$ is arbitrarily small, each variety is supplied at its monopoly price since $\mathcal{T} \to 0$.

The expression (7) shows that allowing consumers to have different ideal horizontal attributes affects the equilibrium markups and prices through the values of the three indices $\mathbb{A}$, $\mathbb{C}$ and $\mathbb{N}$. To be precise, by distributing β we allow for heterogeneous consumers who have different hedonic values for the horizontal attributes of each variety. In contrast, although the standard quadratic utility with constant β does encapsulate horizontal product differentiation, it does so by assuming that consumers have exactly the same hedonic values for the horizontal attributes of varieties.

Last, suppose that the average effective quality $\mathbb{A}/\mathbb{N}$ increases by $\Delta > 0$. Then, if the quality upgrade Δ_s of variety s is such that

$$\Delta_s > \mathcal{T}\Delta$$

then its markup and price will increase, even though the quality upgrade Δ_s may be lower than Δ. In contrast, if the quality upgrade of variety s is smaller than $\mathcal{T}\Delta$, then its markup and price will decrease, even though the quality upgrade Δ_s is positive. Hence, the toughness of competition matters for the determination of the equilibrium markups.

Using the properties of linear demand functions, we readily verify that the equilibrium output of each variety is given by

$$q_s^* = \frac{1}{\beta_s}(p_s^* - c_s) \tag{8}$$

while the corresponding equilibrium operating profits are as follows:

$$\pi_s = \frac{1}{\beta_s}(p_s^* - c_s)^2.$$

These various properties show that our model retains the flexibility displayed by the standard quadratic utility model, while enabling to capture several new effects. In order to gain further insights on the role played by each source of heterogeneity, we now consider the following special cases.

1. When the cost c_s is the only idiosyncratic parameter, firms charge higher prices if and only if they face higher marginal costs:

$$p_s^* = \frac{\alpha + c_s}{2} - \frac{\gamma N}{2\beta + \gamma N}\frac{\alpha - \overline{c}}{2}$$

where

$$\overline{c} = \frac{1}{N}\int_S c_r \mathrm{d}r$$

is the average cost (since β is the same across varieties, we simply have $\mathbb{C} = N\overline{c}/\beta$). Given the linearity of demand functions, firms pass onto their customers half of their costs. This implies that higher-cost firms have lower markups, quantities sold and profits.

Whereas c_s has a negative impact on firm s-profitability, the average cost has a positive impact because increasing $\overline{c}$ relaxes competition. Therefore, only idiosyncratic costs and market indices interact in determining the equilibrium price, markup and output for each variety.

Unfortunately, this cost-based approach to heterogeneity does not provide much flexibility in terms of firms' characteristics. The most evident limits are that: *(i)* higher prices can stem only from higher costs; *(ii)* lower markups always coincide with lower levels of output; *(iii)* the ratio between markup and output is constant and the same across varieties; and *(iv)* firms with identical costs charge the same price. These effects are illustrated in Figure 2.

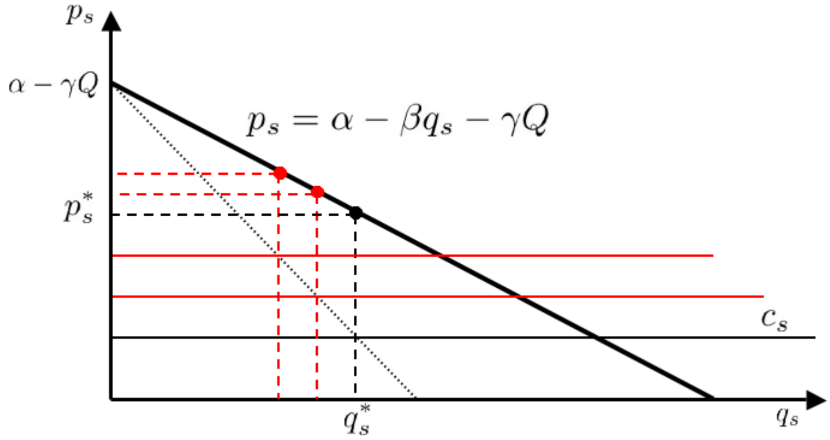

Figure 2: Equilibrium prices and quantities under cost heterogeneity only.

We now show how vertical and horizontal differentiation features address these issues.

2. Once we also allow for heterogeneity in α and c, higher prices need not be the symptom of productive inefficiency: they may also reflect higher quality. Unlike the case of cost heterogeneity alone, the introduction of vertical differentiation allows to have higher prices associate with higher markups and outputs:

$$p_s^* = \frac{\alpha_s + c_s}{2} - \frac{\gamma N}{2\beta + \gamma N} \frac{\overline{\alpha} - \overline{c}}{2}$$

where

$$\overline{\alpha} = \frac{1}{N} \int_S \alpha_r \mathrm{d}r$$

is the average quality. A new feature emerges: market size and productive efficiency are not anymore the only sources of difference in competitive pressures across markets. The average quality of varieties available in a particular market plays a role as well. In particular, it is striking that the average quality $\overline{\alpha}$ affects the price index in such a way that, although markets with higher average quality show higher prices than markets with lower average quality, competition is tougher in the former. That is, idiosyncratic and average qualities work in opposite directions in determining the equilibrium price and markup of a variety. Hence, by introducing heterogeneity in quality, the above-mentioned relationships *(i)* and *(iv)* do not hold anymore.

Notably, this suggests that high quality may turn out to be as important as high productive efficiency in preventing access to a particular market. If products in developed countries have a higher average quality than products in developing countries, these properties may reconcile the Balassa-Samuelson hypothesis, which states that developed economies display higher prices, with the empirical observation that it is difficult for developing country firms to penetrate and thrive in developed countries' markets (Edwards and Lawrence, 2010). See Figure 3 for an illustration of the shifting effect when α is the only source of heterogeneity.

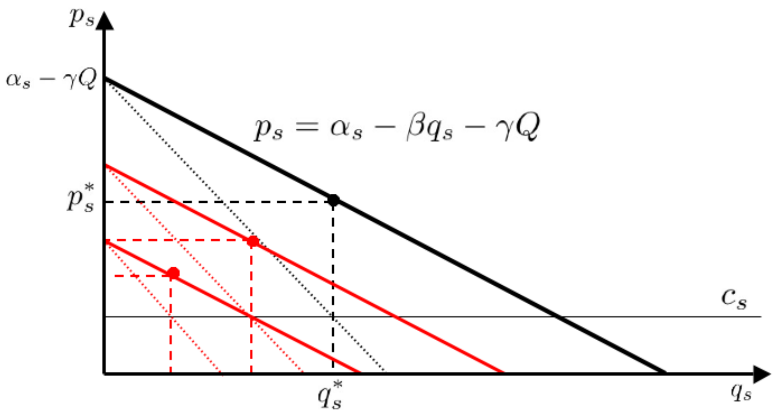

Figure 3: Equilibrium prices and quantities under vertical differentiation only.

3. Last, we account for heterogeneity in β and c only. In this case, the equilibrium price is given by

$$p_s^* = \frac{\alpha + \gamma \frac{\mathbb{C}}{2}}{2 + \gamma \mathbb{N}} + \frac{c_s}{2}$$

which is independent of β_s. Figure 4 illustrates how the market price is determined when β is the only distributed parameter.

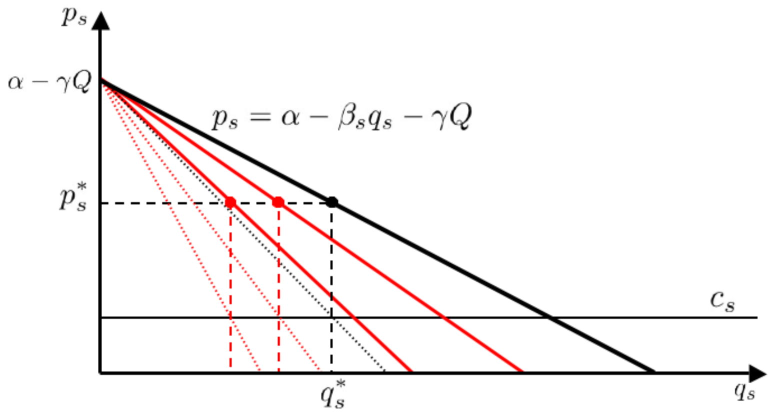

Figure 4: Equilibrium prices and quantities under horizontal differentiation only

Nevertheless, as shown in (8), the ratio between markup and output varies across varieties: if β_s decreases, variety s' markup and price remain constant but its output of increases. In other words, a change in β_s results in output rather than price adjustments, and thus the relationship *(iii)* does not hold anymore. In addition, if the parameter β_s decreases strongly for a small range of varieties in a way such that $\mathbb{C}$ remains more or less constant while $\mathbb{N}$ increases, then prices and markups decrease. Therefore, markups and outputs of the varieties experiencing lower β_s move in opposite directions. As a result, relationship (ii) need not hold either.

4 Conclusions

In this paper we have presented a generalization of the quadratic utility model. By allowing their parameters to be variety- or market-variety-specific, instead of homogeneous within a sector, we show that the utility function they propose can be seen as the aggregation across a mass of varieties of a traditional model of vertical and horizontal differentiation. After having shown how a unique functional form may fit both recent trade models and traditional frameworks of product differentiation, we adapt the results and definitions of the latter, characterized by unitary purchases of mutually exclusive varieties, to a framework more suitable for the study of intra-industry trade, where consumers are allowed to buy different varieties of the same kind of good in different quantities. In equilibrium, vertical attributes of a variety are shown to have a direct effect on prices and sales, whereas horizontal characteristics only affect quantities sold.

The interaction between three sources of demand-side heterogeneity - quality, tastes, substitutability - and the more commonly assumed variability in production costs is shown to have the potential to address the data-fitting issues arisen from the empirical testing of existing intra-industry trade theories. In particular, taste mismatch, which can be directly estimated through markups and quantities, can be used to capture and exploit the vast amount of variability in quantities sold for given levels of prices and markups.

Measurable horizontal differentiation parameters can then be used to obtain new aggregate indices that are closely related to the competitiveness of a particular market. By weighting prices, costs, quality and the mass of firms by this taste-mismatch parameter, we can improve the accuracy of our estimates of competitive interactions between individual varieties and market aggregates.

Disentangling the effects of productive efficiency, substitutability, vertical and horizontal differentiation, the model sheds new light and adds generality to a theoretical framework widely used to study trade patterns and firm dynamics without altering it substantially. The technical innovations proposed have the potential to accommodate puzzling empirical results and reconcile them with theory, providing at the least a series of consistency checks to adopt in future works on intra-industry trade.

Given its full identifiability, the model is ready to be directly tested and confronted with alternative models using the available micro-level data. If proven empirically relevant, it can deepen our understanding of the indirect market interactions between heterogeneous varieties in a sector, helping us better define the determinants of firm performance in different markets and the expected effects of changes in trade policy.

References

Altomonte, C., Colantone, I. and Pennings, E. (2010). International trade with heterogenous firms and asymmetric product varieties. Open access publications, Katholieke Universiteit Leuven.

Anderson, S., Palma, A. and Thisse, J. (1992). *Discrete choice theory of product differentiation*. The MIT Press.

Anderson, S. P. and Neven, D. J. (1989). Market efficiency with combinable products. *European Economic Review* 33: 707–719.

Antoniades, A. (2008). Heterogeneous Firms, Quality, and Trade. mimeo, Columbia University.

Baldwin, R. and Harrigan, J. (2007). Zeros, Quality and Space: Trade Theory and Trade Evidence. NBER Working Papers 13214, National Bureau of Economic Research.

Bernard, A. B. and Jensen, J. B. (1994). Exporters, Jobs and Wages in U.S. Manufacturing: 1976-1987. Working papers 95-7, Massachusetts Institute of Technology (MIT), Department of Economics.

Bernard, A. B., Jensen, J. B. and Schott, P. K. (2006). Survival of the best fit: Exposure to low-wage countries and the (uneven) growth of u.s. manufacturing plants. *Journal of International Economics* 68: 219–237.

Bresnahan, T. F. and Reiss, P. C. (1991). Entry and competition in concentrated markets. *The Journal of Political Economy* 99: 977–1009.

Brooks, E. (2003). Products and Prejudice: Measuring Country-of-Origin Bias in U.S. Wine Imports. Santa Cruz Center for International Economics, Working Paper Series 1000, Center for International Economics, UC Santa Cruz.

Brooks, E. L. (2006). Why don't firms export more? product quality and colombian plants. *Journal of Development Economics* 80: 160–178.

Chung, C. and Song, M. (2008). Preference for Cultural Goods: The Case of Korea Film Market. Georgia institute of technology, mimeo, Georgia Institute of Technology.

Crozet, M., Head, K. and Mayer, T. (2009). Quality Sorting And Trade: Firm-Level Evidence For French Wine. Working Papers 53883, American Association of Wine Economists.

Eaton, J., Eslava, M., Kugler, M. and Tybout, J. (2007). Export Dynamics in Colombia: Firm-Level Evidence. Borradores de economía 446, Banco de la República, Colombia.

Edwards, L. and Lawrence, R. Z. (2010). Do Developed and Developing Countries Compete Head to Head in High-tech? National bureau of economic research working paper series.

Engel, C. and Rogers, J. H. (2000). Deviations from purchasing power parity: causes and welfare costs. International Finance Discussion Papers 666, Board of Governors of the Federal Reserve System (U.S.).

Fajgelbaum, P. D., Grossman, G. M. and Helpman, E. (2009). Income Distribution, Product Quality, and International Trade. NBER Working Papers 15329, National Bureau of Economic Research.

Ferreira, F. and Waldfogel, J. (2010). Pop Internationalism: Has A Half Century of World Music Trade Displaced Local Culture? Working Paper 15964, National Bureau of Economic Research.

Fontagné, L., Gaulier, G. and Zignago, S. (2008). Specialization across varieties and North-South competition. *Economic Policy* 23: 51–91.

Foster, L., Haltiwanger, J. and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review* 98: 394–425.

Friberg, R., Paterson, R. W. and Richardson, A. D. (2010). Why is there a home bias? A case study of Wine. CEPR Discussion Papers 7885, Centre for Economic Policy Research.

Gabszewicz, J. J. and Thisse, J. (1979). Price competition, quality and income disparities. *Journal of Economic Theory* 20: 340–359.

Gaulier, G., Martin, J., Mejean, I. and Zignago, S. (2008). International Trade Price Indices. Working Papers 2008-10, CEPII research center.

Gervais, A. (2008). Product Quality and Trade Liberalization. University of notre dame, mimeo, University of Notre Dame.

Gervais, A. (2010). Product Quality and Firm Heterogeneity in International Trade. mimeo, University of Notre Dame.

Goldberg, P. K. and Verboven, F. (2001). The evolution of price dispersion in the european car market. *Review of Economic Studies* 68: 811–48.

Goldberg, P. K. and Verboven, F. (2005). Market integration and convergence to the law of one price: evidence from the european car market. *Journal of International Economics* 65: 49–73.

Gordon, I. (2010). Price and Non-Price Spatial Competition. Paper for conference on spatial economics and trade, university of strathclyde, mimeo, London School of Economics.

Gorg, H., Halpern, L. and Murakozy, B. (2010). Why do within firm-product export prices differ across markets? Kiel Working Papers 1596, Kiel Institute for the World Economy.

Hallak, J. C. (2004). Product Quality, Linder, and the Direction of Trade. NBER Working Papers 10877, National Bureau of Economic Research.

Hallak, J. C. and Sivadasan, J. (2009). Firms' Exporting Behavior under Quality Constraints. Working Papers 09-13, Center for Economic Studies, U.S. Census Bureau.

Helble, M. and Okubo, T. (2008). Heterogeneous Quality Firms and Trade Costs. Discussion Paper Series 220, Research Institute for Economics & Business Administration, Kobe University.

Hotelling, H. (1929). Stability in competition. *The economic journal* 39: 41–57.

Hummels, D. and Klenow, P. J. (2005). The variety and quality of a nation's exports. *American Economic Review* 95: 704–723.

Hummels, D. and Skiba, A. (2004). Shipping the good apples out? an empirical confirmation of the Alchian-Allen conjecture. *Journal of Political Economy* 112: 1384–1402.

Iacovone, L. and Javorcik, B. S. (2008). Shipping good tequila out: Investment, domestic unit values and entry of multi-product plants into export markets. mimeo, University of Oxford.

Irmen, A. and Thisse, J. (1998). Competition in multi-characteristics spaces: Hotelling was almost right. *Journal of Economic Theory* 78: 76–102.

Johnson, R. C. (2007). Trade and prices with heterogeneous firms. University of california, berkeley, mimeo, University of California, Berkeley.

Katayama, H., Lu, S. and Tybout, J. R. (2009). Firm-level productivity studies: Illusions and a solution. *International Journal of Industrial Organization* 27: 403–413.

Khandelwal, A. (2009). The Long and Short (of) Quality Ladders. NBER Working Papers 15178, National Bureau of Economic Research.

Kneller, R. and Yu, Z. (2008). Quality Selection, Chinese Exports and Theories of Heterogeneous Firm Trade. Discussion Papers 08/44, University of Nottingham, GEP.

Kranich, J. (2007). Too much R&D? - Vertical differentiation in a model of monopolistic competition. Working Paper Series in Economics 59, University of Luneburg, Institute of Economics.

Kugler, M. (2008). Product Quality at the Plant Level: Plant Size, Exports, Output Prices and Input Prices in Colombia. Working Papers eg0058, Wilfrid Laurier University, Department of Economics.

Kugler, M. and Verhoogen, E. A. (2007). Product Quality at the Plant Level: Plant Size, Exports, Output Prices and Input Prices in Colombia. Discussion Papers 0708-12, Columbia University, Department of Economics.

Lederer, P. J. and Hurter, J., Arthur P. (1986). Competition of firms: Discriminatory pricing and location. *Econometrica* 54: pp. 623–640.

Manova, K. and Zhang, Z. (2009). Quality Heterogeneity across Firms and Export Destinations. NBER Working Papers 15342, National Bureau of Economic Research.

Melitz, M. J. (2003). The impact of trade on Intra-Industry reallocations and aggregate industry productivity. *Econometrica* 71: 1695–1725.

Melitz, M. J. and Ottaviano, G. I. P. (2008). Market size, trade, and productivity. *Review of Economic Studies* 75: 295–316.

Neven, D. and Thisse, J.-F. (1989). On Quality And Variety Competition. CORE Discussion Papers 1989020, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).

Ottaviano, G. I. P., Tabuchi, T. and Thisse, J. (2002). Agglomeration and trade revisited. *International Economic Review* 43: 409–436.

Schott, P. K. (2004). Across-product versus within-product specialization in international trade. *The Quarterly Journal of Economics* 119: 646–677.

Schott, P. K. (2008). The relative sophistication of chinese exports. *Economic Policy* 23: 5–49.

Shaked, A. and Sutton, J. (1982). Relaxing price competition through product differentiation. *Review of Economic Studies* 49: 3–13.

Tirole, J. (1988). *The Theory of Industrial Organization*, MIT Press Books *1*. The MIT Press.

Verhoogen, E. A. (2008). Trade, quality upgrading, and wage inequality in the mexican manufacturing sector. *The Quarterly Journal of Economics* 123: 489–530.

Vives, X. (2001). *Oligopoly Pricing: Old Ideas and New Tools*. No. 026272040x in MIT Press Books. The MIT Press.